Module Overview

In this inquiry module, students develop an understanding of some of the terms and postulates that form the logical basis of geometry. Students discuss precise nature of mathematical definitions and their necessity in deductive reasoning. They apply reasoning to complete geometric constructions and explain why they work.

Essential Questions

- What criteria are used when selecting a tool?
- How can "if-then" statements be logically applied?

Student Focal Points

1) Understanding the precise nature and application of mathematical definitions and "if-then" statements
2) Making and justifying geometric constructions

Standards for Mathematical Practice

Mathematically proficient students...

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Content

Sense-Making Concepts Standard(s)	Sense-Making Strategies Standard(s)	Sense-Making Applications/Modeling Standard(s)
G.CO.1 G.CO.12	7.G.5	

Module A1 - Scope & Sequence

Duration	Standard(s)	
2 days	G.CO.1	**Lesson 1: Undefined Terms: Point, Line, Plane, & Space** MP#6
	Text: Section 1.3 (Points, Lines, and Planes) Students need to develop a foundational understanding in notation and spatial reasoning between dimensions. Teacher Resource(s): • The Role of Mathematical Definitions in Mathematics and in Undergraduate Mathematics Courses (pages 223-225) Collaborative Activity/Task: Drawing Geometric Figures Tools/Technology: • Possible instructional strategies: Venn Diagram, Frayer Model	
2-3 days	G.CO.1 G.CO.12	**Lesson 2: Segment Addition and Congruence** MP#5, 7
	Launching Lesson: **MP6 – Attend to precision** Teacher Resource(s): ○ Common Core State Standards for Mathematics, 2010. (pages 6-8) Tools/Technology: ○ Possible instructional strategies: Anchor Chart	
	Text: Section 1.5 (Segments & their Measures) Constructions should focus on conceptual understanding and argument, rather than solely on procedure. Teacher Resource(s): • Math Open Reference: Copying a Line Segment • Brightstorm (YouTube): Duplicating a Line Segment Collaborative Activity/Task: Compass Construction Activity – Copy a Segment Tools/Technology: • Compass-&-Straightedge Construction (Geometry software will be used in a later lesson. Students should focus on the conceptual understanding related to "by-hand" constructions at this time.)	
2-3 days	G.CO.1 G.CO.12	**Lesson 3: Angle Addition and Congruence** MP#5, 6, 7
	Text: Section 1.6 (Angles & their Measures) Constructions should focus on conceptual understanding and argument, rather than solely on procedure. Teacher Resource(s): • Math Open Reference: Copying an Angle • Brightstorm (YouTube): Duplicating an Angle Collaborative Activity/Task: Compass Construction Activity – Copy an Angle Tools/Technology: • Compass-&-Straightedge Construction (Geometry software will be used in a later lesson. Students should focus on the conceptual understanding related to "by-hand" constructions at this time.)	
5 days	7.G.5	**Lesson 4: Angle Pairs** MP#6, 7
	Text: • Section 2.3 (Complementary and Supplementary Angles)	

	• Section 2.4 (Vertical Angles) • Section 3.1 (Relationships Between Lines) Teacher Resource(s): Collaborative Activity/Task: Angles and Intersection Lines Tools/Technology: • Students will investigate the relationships between angles created by two intersecting lines using a protractor	
1-2 days	G.CO.12	**Lesson 5: Segment Bisectors** MP #5, 6, 7
	Text: Section 2.1 (Segment Bisectors) Constructions should focus on conceptual understanding and argument, rather than solely on procedure. Teacher Resource(s): • Brightstorm (YouTube): Midpoints and Congruent Segments • eHow (YouTube): How to Construct the Midpoint of a Segment for Geometry • Math Open Reference: Perpendicular Bisector of Line Segment Collaborative Activity/Task: Compass Construction Activity – Bisect a Segment Tools/Technology: • Paper-folding Construction and Compass-&-Straightedge Construction (Geometry software will be used in a later lesson. Students should focus on the conceptual understanding related to "by-hand" constructions at this time.)	
1-2 days	G.CO.12	**Lesson 6: Angle Bisectors** MP #5, 6, 7
	Text: Section 2.2 (Angle Bisectors) Constructions should focus on conceptual understanding and argument, rather than solely on procedure. Teacher Resource(s): • Math Open Reference: Bisecting an Angle • Brightstorm (YouTube)– Angle Bisector • Angle Bisector Construction (YouTube) Collaborative Activity/Task: Compass Construction Activity – Bisect an Angle Tools/Technology: • Paper-Folding Construction and Compass-&-Straightedge Construction (Geometry software will be used in a later lesson. Students should focus on the conceptual understanding related to "by-hand" constructions at this time.)	
1-2 days	G.CO.12	**Lesson 7: Constructions** MP #1, 5
	Text: N/A Teacher Resource(s): • GeoGebra – GBAPS Resource Collaborative Activity/Task: Tools/Technology: • Compass-&-Straightedge Construction • Dynamic Online Graphing software: GeoGebra	
2 days		Module A1 Review **Module A1 Test** (common assessment)

Total Days: 16-21 days

Practice Standards Unpacking

MP#1: Mathematically proficient students use appropriate tools strategically.

Summary:

- Use available tools recognizing the strengths and limitations of each.
- Use estimation and other mathematic knowledge to detect possible errors.
- Identify relevant external mathematical resources to pose and solve problems.
- Use technological tools to deepen their understanding of mathematics.

Questions to Develop Mathematical Thinking:

- What mathematical tools could we use to visualize and represent the situation?
- What information do you have?
- What do you know that is not stated in the problem?
- What approach are you considering trying first?
- What estimate di you make for the solution?
- In this situation would it be helpful to use… a graph, number line…, ruler…, diagram…, calculator…, manipulative?
- Why was it helpful to use…?
- What can suing a _____ show us that _____ may not?
- In what situations might it be more informative or helpful to use…?

High School Flip Book
Common Core State Standards for Mathematics
http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

MP#6: Mathematically proficient students attend to precision.

Summary:

- Communicate precisely with other and try to use clear mathematical language when discussing their reasoning.
- Understand meanings of symbols used in mathematics and can label quantities appropriately.
- Express numerical answers with a degree of precision appropriate for the problem context.
- Calculate efficiently and accurately.

Questions to Develop Mathematical Thinking:

- What mathematical terms apply in this situation?
- How did you know your solution was reasonable?
- Explain how you might show that your solution answers the problem.
- Is there a more efficient strategy?
- How are you showing the meaning of the quantities?
- What symbols or mathematical notations are important in this problem?
- What mathematical language…, definitions…, properties… can you use to explain…?
- How could you test your solution to see if it answers the problem?

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Content Standards Unpacking

7.G.5 (Sense-Making Strategies Standard)

Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Evidence of Student Attainment: Students…

- Given multi-step problems involving angle measures, use knowledge of supplementary, complementary, vertical, and adjacent angles to create and solve equations for unknown angles, and justify solutions and solution paths.

Teacher Vocabulary: Supplementary angles, complementary angles, vertical angles, adjacent angles

Knowledge: Students know…

- Defining characteristics of, relationships among, and situations that produce supplementary, complementary, vertical, and adjacent angles.
- Strategies for visually representing contexts involving angle measures.

Skills: Students are able to…

- Visually represent verbal contexts involving angles.
- Strategically choose and apply appropriate methods for representing and calculating angle measures.
- Use logical reasoning to apply knowledge of supplementary, complementary, vertical, and adjacent angles to create equations and solve multi-step problems.

Understanding: Students understand that…

- Angle measure is additive.
- Angles created by two intersecting lines have relationships that can be used to solve problems.

Vocabulary for Student Discourse

Supplementary angles	Complementary angles	Vertical angles
Adjacent angles	Intersecting	Right angle

Notation: ° (degrees) $m\angle 1 =$ "measure of angle 1"
$m\angle 1 + m\angle 2 = m\angle ABC$

Related Standards: 6.EE.6, 7.EE.4, 8.G.5, G.CO.9, G.CO.10, G.SRT.7, G.C.3, G.GPE.7

Further Discussion & Illustrations of Standard 7.G.5:

G.CO.1 (Sense-Making Concepts Standard)

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Evidence of Student Attainment: Students…

Given undefined notation of point, line, distance along a line, and distance around a circular arc,

- Develop precise definitions of angle, circle, perpendicular line, parallel line, and line segment.
- Identify examples and non-examples of angles, circles, perpendicular lines, parallel lines, and line segments.

Teacher Vocabulary:

Knowledge: Students know…

- Undefined notations of point, line, distance along a line, and distance around a circular arc.
- Properties of a mathematical definition, i.e., the smallest amount of information and properties that are enough to determine the concept. (Note: may not include all information related to concept).

Skills: Students are able to…

- Use known and developed definitions and logical connections to develop new definitions.

Understanding: Students understand that…

- Geometric definitions are developed from a few undefined notions by a logical sequence of connections that lead to a precise definition.
- A precise definition should allow for the inclusion of all examples of the concept, and require the exclusion of all non-examples.

Vocabulary for Student Discourse

Angle	Circle	Perpendicular lines
Parallel lines	Line segment	Circular arc
Arc length	Subtend	Point
Line		

Notation:

"Angle" $= \angle$	"Parallel" $= \parallel$
"Perpendicular" $= \perp$	"Line AB" $= \overleftrightarrow{AB}$
"Segment AB" $= \overline{AB}$	"Length of Segment AB" $=$ AB
"Distance between points A & B" $=$ AB	"Circular Arc AB" $= \overset{\frown}{AB}$
"Measure of Arc AB" $= m\overset{\frown}{AB}$	"Measure of Angle ACB" $= m\angle ACB$

Related Standards:

Further Discussion & Illustrations of Standard G.CO.1:

G.CO.12 (Sense-Making Concepts Standard)

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on a line.

Evidence of Student Attainment: Students…

- Make and justify formal geometric constructions with a variety of tools and methods (e.g., compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.()including the following: Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a liner parallel to a given line through a point not on the line.
- Compare and contrast different methods for doing the same construction, and identify geometric properties that justify steps in the constructions.

Teacher Vocabulary:

Knowledge: Students know…

- Methods for accurately using tools to perform geometric constructions, including compass and straightedge, string, reflective devices, paper folding, and dynamic geometric software.
- Methods for justifying a geometric construction using geometric properties.

Skills: Students are able to…

- Choose and use appropriate construction tools strategically to perform geometric constructions.
- Use logical reasoning and properties of and between geometric figures to justify geometric constructions.

Understanding: Students understand that…

- Limiting oneself to a specific tool or set of tools to perform a geometric construction illuminates important mathematical features of the object being constructed.
- Different tools for geometric constructions may offer different levels of precision in the construction and the purpose of the construction should help determine the tool the choice.
- Properties of geometric figures can and should be used to verify the correctness of geometric constructions regardless of the construction tool or method used.

Vocabulary for Student Discourse

Geometric Construction	Segment Bisector	Angle Bisector
Perpendicular Bisector	Midpoint	

Notation:

Related Standards: 7.G.2

Further Discussion & Illustrations of Standard G.CO.12: