Module Overview

In this inquiry module, students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry.

Essential Questions

- Why is proof important?
- How do we argue that transformations preserve shape?
- How do proportions help us to solve problems?

Student Focal Points

1) Understanding similarity in terms of dilations and rigid motions
2) Proving theorems involving similarities
3) Defining trigonometric ratios, finding relationships among them, and solving problems involving right triangles in context
4) Applying trigonometric ratios in modeling situations

Standards for Mathematical Practice

Mathematically proficient students...

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. **Look for and make use of structure.**
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Content

Sense-Making Concepts Standard(s)	Sense-Making Strategies Standard(s)	Sense-Making Applications/Modeling Standard(s)
8.G.3 G.SRT.3 G.SRT.4 G.SRT.6 G.SRT.7	G.SRT.1 G.SRT.2	G.SRT.1 G.SRT.4 G.SRT.5 G.SRT.8 G.MG.1 G.MG.2 G.MG.3

Module B - Scope & Sequence

Duration	Standard(s)	
2-3 days	8.G.3 G.SRT.1	**Lesson 1: Dilations** MP#4, 5, 7
	Text: Section 7.6 (Dilations) Dilations should be done on and off the coordinate plane and with GeoGebra. Teacher Resource(s): Collaborative Activity/Task: 1) Guided Instructions to introduce the lesson 2) Coordinate dilations with center not on origin Tools/Technology: • GeoGebra can be used for the Coordinate dilations activity	
4-5 days	G.SRT.2 G.SRT.3 G.MG.3	**Lesson 2: Similar Polygons and Triangles** MP#2, 3, 4
	Text: Section 7.2 (Similar Polygons), 7.3 (Showing Triangles are Similar: AA), 7.4 (Showing Triangles are Similar: SSS and SAS) Teacher Resource(s): Collaborative Activity/Task: 1) Introductory Activity for Similar Polygons 2) Collaborative Activity for AA Similarity 3) Compare/Contrast Summary Activity Tools/Technology:	
3 days	G.SRT.4 G.SRT.5	**Lesson 3: Proportions in Similar Shapes** MP#1, 7
	Text: Section 7.5 (Proportions and Similar Triangles) Teacher Resource(s): Collaborative Activity/Task: Tools/Technology: Use GeoGebra to introduce proportionality using parallel lines and measurements	
1-2 days	G.SRT.4	**Lesson 4: Proving Pythagorean Theorem using Similar Figures** MP#2, 3, 5
	Text: no text section for this lesson Teacher Resource(s): **http://www.math.nmsu.edu/breakingaway/Lessons/PTUST/PTUST.html** **http://www.youtube.com/watch?v=Ng2EpkKooo4&safety_mode=true&persist_safety_mode=1** **https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-geometry/cc-8th-pythagorean-proofs/v/pythagorean-theorem-proof-using-similarity** Collaborative Activity/Task: Tools/Technology:	
3 days	G.SRT.6 G.SRT.7	**Lesson 5: Trigonometric ratios** MP #6, 8
	Text: Section 10.4 (Tangent Ratios), 10.5 (Sine and Cosine Ratios) Emphasis on trigonometric ratios based on similar triangles. Teacher Resource(s): Collaborative Activity/Task: Introductory Collaborative Activity to generate the ratios Tools/Technology: Into. Activity can be done on GeoGebra	

3 days	G.SRT.8 G.MG.1	**Lesson 6: Solving Right Triangles** MP #4, 5, 7	
	Text: Section 10.6 (Solving Right Triangles) Teacher Resource(s): Collaborative Activity/Task: 1) Guided Instruction questions 2)Collaborative Activity solving right triangles 3) Exit slip idea Tools/Technology:		
2 days		Module B Review **Module B Test** (common assessment)	

Total Days: 20-23 days

Content Standards Unpacking

8.G.3 (Sense-Making Concepts Standard)

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Evidence of Student Attainment: Students…

Given a variety of sequences of dilations, translations, rotations, and reflections

- Use a coordinate plane to model and describe the effects of the transformational geometry sequences on given shapes and the corresponding coordinates.
- Compare the properties of the original figure to the newly-created figures to determine similarity and congruence.

Teacher Vocabulary: translations, rotations, reflections, coordinates, dilations

Knowledge: Students know…

- Methods of modeling the effects of rotations, reflections, and translations of 2-D figures on coordinate planes.

Skills: Students are able to…

- Accurately perform dilations, rotations, reflections, and transformations on objects in the coordinate plane.
- Communicate the results of transformational geometry on objects and their corresponding coordinates in the coordinate plane.

Understanding: Students understand that…

- The x,y-coordinates will be directly affected by the dilations, rotations, reflections, and translations that map one object on the another.

Vocabulary for Student Discourse

pre-image	image	translation
rotation	center of rotation	reflection
line of reflection	coordinate	ordered pair
dilation	transformation	transform
similar	congruent	corresponding parts

Notation: $\cong$ $\sim$ $\angle$

Related Standards: 6.NS.6, 6.G.3, 7.G.1, F.BF.3, G.CO.5, G.SRT.1

Further Discussion & Illustrations of Standard 8.G.3:

G.SRT.1 (Sense-Making Strategies Standard)

Verify experimentally the properties of dilations given by a center and a scale factor:

a. **A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.**

b. **The dilation of a line segment is longer or shorter in the ratio given by the scale factor.**

Evidence of Student Attainment: Students…

Given a center of dilation, a scale factor, and a polygonal image,

- Create a new image by extending a line segment from the center of dilation through each vertex of the original figure by the scale factor to find each new vertex.
- Present a convincing argument that line segments created by the dilation are parallel to their pre-images unless the pass through the center of dilation, in which case they remain on the same line.
- Find the ratio of the length of the line segment from the center of dilation to each vertex in the new image and the corresponding segment in the original image and compare that ratio to the scale factor.
- Conjecture a generalization of these results for all dilations.

Teacher Vocabulary: dilations, center, scale factor

Knowledge: Students know…

- Methods for finding the length of line segments (both in a coordinate plane and through measurement).
- Dilations may be performed on polygons by drawing lines through the center of dilation and each vertex of the polygon then marking off a line segment changed from the original by the scale factor.

Skills: Students are able to…

- Accurately create a new image from a center of dilation, a scale factor, and an image.
- Accurately find the length of line segments and ratios of line segments.
- Communicate with logical reasoning a conjecture of generalization from experimental results.

Understanding: Students understand that…

- A dilation uses a center and line segments through vertex points to create an image which is similar to the original image but in a ratio specified by the scale factor.
- The ratio of the line segment formed from the center of dilation to a vertex in the new image and the corresponding vertex in the original image is equal to the scale factor.

Vocabulary for Student Discourse

dilation	center	scale factor
ratio	pre-image	image
vertex	corresponding parts	

Notation: "scale factor" = k

Related Standards: 7.G.1, 8.G.3, 8.G.4

Further Discussion & Illustrations of Standard G.SRT.1:

G.SRT.2 (Sense-Making Strategies Standard)

Given two figures, use the definition of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Evidence of Student Attainment: Students…

- Given two figures, determine if they are similar by demonstrating whether one figure can be obtained from the other though a dilation and a combination of translations, reflections, and rotations.
- Given a triangle,
  - Produce a similar triangle through a dilation and a combination of translations, rotations, and reflections.
  - Demonstrate that a dilation and a combination of translations, reflections, and rotations maintain the measure of each angle in the triangles and all corresponding paris of sides of the triangles are proportional.

Teacher Vocabulary: Similarity transformation

Knowledge: Students know…

- Properties of rigid motions and dilations.
- Definition of similarity in terms of similarity transformations.
- Techniques for producing images under a dilation and rigid motions.

Skills: Students are able to…

- Apply rigid motion and dilation to a figure.
- Explain and justify whether or not one figure can be obtained from another through a combination of rigid motion and dilation.

Understanding: Students understand that…

- A figure that may be obtained from another through a dilation and a combination of translations, reflections, and rotations is similar to the original.
- When a figure is similar to another the measures of all corresponding angles are equal, and all of the corresponding sides are in the same proportion.

Vocabulary for Student Discourse

similarity transformation	dilation	reflection
rotation	translation	similar figures
rigid motion	corresponding pairs of angles	corresponding pairs sides
proportionality		

Notation: ~

Related Standards:

Further Discussion & Illustrations of Standard G.SRT.2:

G.SRT.3 (Sense-Making Concepts Standard)

Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Evidence of Student Attainment: Students…

Given two triangles,

- Explain why if the measures of two angles from one triangle are equal to the measures of two angles from another triangle, then measures of the third angles must be equal to each other.
- Use this established fact and the properties of a similarity transformation to demonstrate that the Angle-Angle (AA) criterion for similar triangle is sufficient.

Teacher Vocabulary: AA criterion

Knowledge: Students know…

- The sum of the measures of the angles of a triangle is 180 degrees.
- Properties of rigid motions and dilations.

Skills: Students are able to…

- Explain and justify why the third pair of corresponding angles of two triangles must be equal if each of the other two corresponding pairs are equal.
- Justify through the use of rigid motion and dilation why corresponding sides of triangles are in the same proportion if the measures of two pairs of corresponding angles are equal.

Understanding: Students understand that…

- It is beneficial to have minimal sets of requirements to justify geometric results (i.e., use AA instead of all sides proportional and all angles congruent for similarity).
- If the measures of two angles of one triangle are equal to the measures of two angles of another triangle, then the triangles a re similar and the similarity of the triangles can be justified through similarity transformations.

Vocabulary for Student Discourse

similar triangles AA Criterion

Notation: $\Delta ABC \sim \Delta DEF$

Related Standards: 8.G.5

Further Discussion & Illustrations of Standard G.SRT.3:

G.SRT.4 (Sense-Making Concepts & Applications/Modeling Standard)

Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

Evidence of Student Attainment: Students…

- Given a triangle and a line parallel to one of the sides, prove the other two sides are divided proportionally by using AA, similarity properties, previously proved theorems and properties of equality (Table 4).
- Given a triangle with two of the sides divided proportionally, prove the line dividing the sides is parallel to the third side of the triangle.
- Given a right triangle, use similar triangles and properties of equality (Table 4) to prove the Pythagorean Theorem.

Teacher Vocabulary: theorem

Knowledge: Students know…

- Properties of similar triangles and methods of showing that triangles are similar.
- Properties of equality (Table 4).
- Previously proven theorems including those concerning parallel lines.

Skills: Students are able to…

- Apply properties of similar triangles to justify relationships of the sides of a triangle.
- Explain and justify that a line passing through the triangle divides the sides proportionally, if and only if, the line is parallel to a side of the triangle.
- Justify the Pythagorean Theorem through the use of similar triangles.

Understanding: Students understand that…

- Triangle similarity may be used to justify theorems involving the connection between the proportion of sides and wther or not the line dividing the sides is parallel to the other side of the triangle.
- Through the use of similar triangles, a right triangle may be divided into two right triangles which are similar to the original right triangle; therefore, the corresponding sides must be proportional and may be used to prove the Pythagorean Theorem.
- The same theorem may be proven in many different ways (i.e., the Pythagorean Theorem).

Vocabulary for Student Discourse

theorem Pythagorean Theorem proof

Notation:

Related Standards: 8.G.6

Further Discussion & Illustrations of Standard G.SRT.4:

G.SRT.5 (Sense-Making Applications/Modeling Standard)

Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Evidence of Student Attainment: Students…

Given a contextual situation involving triangles,

- Determine solutions to problems by applying congruence and similarity criteria for triangles to assist in solving the problem.
- Justify solutions and critique the solutions of others.

Given a geometric figure,

- Establish and justify relationships in the figure through the use of congruence and similarity criteria for triangles .

Teacher Vocabulary: congruence and similarity criteria for triangles

Knowledge: Students know…

- Criteria for congruent (SAS, ASA, AAS, SSS) and similar (AA) triangles and transformation criteria.
- Techniques to apply criteria of congruent and similar triangles for solving a contextual problem.

Skills: Students are able to…

- Accurately solve a contextual problem by applying the criteria of congruent and similar triangles.
- Provide justification for the solution process.
- Analyze the solutions of others and explain why their solutions are valid or invalid.
- Justify relationships in geometric figures through the use of congruent and similar triangles.

Understanding: Students understand that…

- Congruence and similarity criteria for triangles may be used to find solutions of contextual problems.
- Relationships in geometric figures may be proven through the use of congruent and similar triangles.

Vocabulary for Student Discourse

congruence criteria for triangles

similarity criteria for triangles

corresponding parts

Notation:

Related Standards: 8.G5

Further Discussion & Illustrations of Standard G.SRT.5:

G.SRT.6 (Sense-Making Concepts Standard)

Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Evidence of Student Attainment: Students…

Given a collection of right triangles,

- Construct similar right triangles of various sizes for each right triangle given.
- Compare the ratios of the sides of the original triangles to the ratios of the sides of the similar triangles.
- Communicate observations made about changes (or not change) to such ratios as the length of the side opposite an angle to the hypotenuse, or the side opposite the angle to the side adjacent, as the size of the angle changes or in the case of similar triangles, remains the same.
- Summarize these observations by defining the six trigonometric ratios.

Teacher Vocabulary: side ratios, trigonometric ratios

Knowledge: Students know…

- Techniques to construct similar triangles.
- Properties of similar triangles.

Skills: Students are able to…

- Accurately find the side ratios of triangles.
- Explain and justify relationship between the side ratios of a right triangle and the angles of a right triangle.

Understanding: Students understand that…

- The ratios of the sides of right triangles are dependent on the size of the angles of the triangle.

Vocabulary for Student Discourse

similar triangles	AA Criterion	
side ratios	trigonometric ratios	cosine
tangent	sine	secant
cosecant	cotangent	adjacent
opposite	hypotenuse	

Notation: sin cos tan csc sec cot

Related Standards: 8.G.7

Further Discussion & Illustrations of Standard G.SRT.6:

G.SRT.7 (Sense-Making Concepts Standard)

Explain and use the relationship between the sine and cosine of complementary angles.

Evidence of Student Attainment: Students…

Given a right triangle,

- Explain why the two smallest angles must be complements.
- Compare the side ratios of opposite/hypotenuse and adjacent/hypotenuse for each of these angles and discuss conclusions.

Given a contextual situation involving right triangles,

- Compare solutions to the situation using the sine of the given angle and the cosine of its complement.

Teacher Vocabulary: sine, cosine, complementary

Knowledge: Students know…

- Methods for finding sine and cosine ratios in a right triangle (e.g., use of triangle properties: similarity; Pythagorean Theorem; isosceles and equilateral characteristics for 45-45-90 and 30-60-90 triangles and technology for others).

Skills: Students are able to…

- Accurately solve a contextual problem by using the sine and cosine ratios.
- Justify solutions and discuss other possible solutions through the use of complementary angles and the sine or cosine ratios.

Understanding: Students understand that…

- The sine of an angle is equal to the cosine of the complement of the angle.
- Switching between using a given angle or its complement and between sine or cosine ratios may be used when solving contextual problems.

Vocabulary for Student Discourse

sine cosine complementary angles

Notation:

Related Standards: 7.G.5, F.TF.1, F.TF.2

Further Discussion & Illustrations of Standard G.SRT.7:

G.SRT.8 (Sense-Making Concepts Standard)

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Evidence of Student Attainment: Students…

Given a contextual situation involving right triangles,

- Create a drawing to model the situation.
- Find the missing sides and angles using trigonometric ratios and the Pythagorean Theorem.
- Use the above information to interpret results in the context of the situation.

Teacher Vocabulary:

Knowledge: Students know…

- Methods of using the trigonometric ratios to solve for sides or angle s in a right triangle.
- The Pythagorean Theorem and its use in solving for unknown parts of a right triangle.

Skills: Students are able to…

- Create an accurate diagram to model a contextual situation involving right triangles and use it to solve the right triangles.
- Identify the trigonometric ratio useful to solve for a particular unknown part of a right triangle and use that ratio to accurately solve for the unknown part.
- Use the Pythagorean Theorem to find unknown sides of a right triangle.
- Explain the solution in terms of the given contextual situation.

Understanding: Students understand that…

- Unknown parts of right triangles may be found through the use of trigonometric ratios, Pythagorean Theorem, or a combination of both.
- Right triangles may be used to model and solve contextual situations.

Vocabulary for Student Discourse

solve a right triangle inverse trigonometric ratio inverse sine

inverse cosine inverse tangent

Notation: $\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$

Related Standards: 8.G.7, F.TF.2, F.TF.8

Further Discussion & Illustrations of Standard G.SRT.8:

G.MG.1 (Sense-Making Applications/Modeling Standard)

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Evidence of Student Attainment: Students…

Given a real-world object,

- Select an appropriate geometric shape to model the object.
- Provide a description of the object through the measures and properties of the geometric shape which is modeling the object.
- Explain and justify the model which was selected.

Teacher Vocabulary:

Knowledge: Students know…

- Techniques to find measures of geometric shapes.
- Properties of geometric shapes.

Skills: Students are able to…

- Model a real-world object through the use of a geometric shape.
- Justify the model by connecting its measures and properties to the object.

Understanding: Students understand that…

- Geometric shapes may be used to model real-world objects.
- Attributes of geometric figures help us identify the figures and find their measures, therefore matching these figures to real-world objects allows the application of geometric techniques to real-world problems.

Vocabulary for Student Discourse

model (physical)

Notation:

Related Standards: 7.G.1, 7.G.6, 8.G.9

Further Discussion & Illustrations of Standard G.MG.1:

G.MG.3 (Sense-Making Applications/Modeling Standard)

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Evidence of Student Attainment: Students…

Given a contextual situation involving design problems,

- Create a geometric method to model the situation and solve the problem.
- Explain and justify the model which was created to solve the problem.

Teacher Vocabulary: geometric methods, design problems

Knowledge: Students know…

- Properties of geometric shapes.
- Characteristics of a mathematical model.

Skills: Students are able to…

- Accurately model and solve a design problem.
- Justify how their model is an accurate representation of the given situation.

Understanding: Students understand that…

- Design problems may be modeled with geometric methods.
- Geometric models may have physical constraints.
- Models represent the mathematical core of a situation without extraneous information, for the benefit in a problem solving situation.

Vocabulary for Student Discourse

Notation:

Related Standards: 7.G.1, 7.G.2, F.IF.5

Further Discussion & Illustrations of Standard G.MG.3: